Maximum likelihood estimation-based denoising of magnetic resonance images using restricted local neighborhoods

Jeny Rajan1, Ben Jeurissen1, Marleen Verhoye2, Johan Van Audekerke2 and Jan Sijbers1

1 Vision Lab, University of Antwerp, Belgium
2 Bio-Imaging Lab, University of Antwerp, Belgium

E-mail: jeny.rajan@ua.ac.be

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Abstract

In this paper, we propose a method to denoise magnitude magnetic resonance (MR) images, which are Rician distributed. Conventionally, maximum likelihood methods incorporate the Rice distribution to estimate the true, underlying signal from a local neighborhood within which the signal is assumed to be constant. However, if this assumption is not met, such filtering will lead to blurred edges and loss of fine structures. As a solution to this problem, we put forward the concept of restricted local neighborhoods where the true intensity for each noisy pixel is estimated from a set of preselected neighboring pixels. To this end, a reference image is created from the noisy image using a recently proposed nonlocal means algorithm. This reference image is used as a prior for further noise reduction. A scheme is developed to locally select an appropriate subset of pixels from which the underlying signal is estimated. Experimental results based on the peak signal to noise ratio, structural similarity index matrix, Bhattacharyya coefficient and mean absolute difference from synthetic and real MR images demonstrate the superior performance of the proposed method over other state-of-the-art methods.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Data acquired by an MRI system are inherently corrupted by statistical noise of which the primary sources are electronic and dielectric and inductive coupling to the conducting solution inside the body (Cárdenas-Blanco et al 2008). Noise remains one of the main causes of quality deterioration in MRI and is a subject in a large number of papers in MRI literature, e.g.
Edelstein \textit{et al} (1984), Henkelman (1985), McVeigh \textit{et al} (1985), Bernstein \textit{et al} (1989), Gudbjartsson and Patz (1995), Macovski (1996). Other than visual analysis, processing techniques such as segmentation, registration or tensor estimation in diffusion tensor MRI (DT-MRI) will be affected or biased due to noise (Jones and Basser 2004, Aja-Fernández \textit{et al} 2010).

Noise can be naturally minimized by averaging images after multiple acquisitions. This, however, may not be feasible in clinical and small animal MRI where there is an increasing need for speed (Manjón \textit{et al} 2008). Thus, post-processing techniques to remove noise in the acquired data are important.

Several filtering techniques to improve the quality of the magnetic resonance (MR) images have been proposed in the literature. Most of the methods proposed earlier can be mainly classified as either based on partial differential equations (PDEs), wavelets or nonlocal means (NLM). A PDE-based approach for filtering MRI was first attempted by Gerig \textit{et al} (1992). In their work, they demonstrated that anisotropic diffusion is an effective filtering technique for MRI in the sense that it can significantly decrease the image noise and simultaneously preserve fine details in the image. A major drawback of their method, however, was the incorrect assumption about the noise distribution. The noise was assumed to be Gaussian instead of Rician, as a result of which a bias is introduced in the filtered image. Such a bias becomes particularly important in low SNR MR images, such as diffusion weighted images (Aja-Fernández \textit{et al} 2008). To account for the Rice distribution, an adaptive anisotropic diffusion method for magnitude MR data was proposed by Sijbers \textit{et al} (1999). Finally, Samsonov and Johnson (2004) presented a noise adaptive nonlinear diffusion filtering technique to denoise MR images with spatially varying noise levels.

All aforementioned PDE methods are based on classical second order Perona–Malik (Perona and Malik 1990) anisotropic diffusion. Although such methods are effective in denoising images, they tend to cause staircase effects in the filtered images (You and Kaveh 2000). To reduce this effect, a noise removal algorithm for MRI based on fourth order PDE was suggested by Lysaker \textit{et al} (2003). The main strength of this method is its ability to process signals with a smooth change in the intensity value. Basu \textit{et al} (2006) used a data likelihood term combined with Perona–Malik anisotropic diffusion to effectively denoise an MR image. Recently Krissian and Aja-Fernández (2009) proposed a noise-driven anisotropic diffusion filter for denoising MR images, in which the diffusion is controlled by the local statistics in the image derived from the linear minimum mean square error (LMMSE) estimator for the Rician model.

A second class of noise filtering schemes are wavelet based (Nowak 1999, Wood and Johnson 1999, Pizurica \textit{et al} 2003). These algorithms exploit the decorrelating properties of the wavelet transform to suppress noise coefficients using statistical inference. Among these methods, Nowak (1999) got much attention, in which a bias removal was proposed, based on the observation that the squared Rician distributed data exhibits a chi-squared distribution with two degrees of freedom. However, in Manjón \textit{et al} (2008), it is mentioned that the aforementioned wavelet-based filters may introduce characteristic artifacts that can be quite problematic. A trilateral filter was proposed in Wong and Chung (2004) to take into account the local structure in the image, in addition to intensity and geometric features. Recently, Delakis \textit{et al} (2007) proposed a wavelet-based denoising algorithm for images acquired with parallel MRI.

During the past few years, NLM-based denoising methods have gained much popularity (Buades \textit{et al} 2005). Manjón \textit{et al} (2008) were the first to attempt MRI denoising with the NLM approach. Coupé \textit{et al} (2008) proposed an optimized blockwise version of the NLM algorithm for denoising MR images. Wiest-Daesselé \textit{et al} (2008) suggested an adaptive NLM

In this paper, we propose a method to denoise magnitude MRI based on the ML estimation method using a restricted local neighborhood. ML estimation can be applied locally (also referred as local ML (LML)) or nonlocally as proposed by He and Greenshields (2009), in which ML estimation is applied on a set of pixels selected based on the similarity of the neighborhood. One drawback of the nonlocal ML (NLML) estimation method is the use of a fixed sample size for the ML estimation, which can cause either under- or over-smoothing. The disadvantage of the LML method is the blurring of edges and the distortion of fine structures in the image (He and Greenshields 2009). This is because the assumption that the signal in the selected small neighborhood is constant is generally not valid. In this paper, we put forward the concept of a restricted local neighborhood as a solution to this problem.

This paper is organized as follows. Section 2 gives a short overview about the noise characteristics in MRI. Section 3 elaborates the proposed method. Section 4 presents the experimental results, comparative evaluation and discussion. Finally, conclusions and remarks are drawn in section 5.

2. Noise characteristics in MRI

The raw, complex MR data acquired in the Fourier domain are characterized by a zero mean Gaussian probability density function (PDF). After the inverse Fourier transform, the noise distribution in the real and imaginary components will still be Gaussian due to the linearity and the orthogonality of the Fourier transform. However, due to the subsequent transform to a magnitude image, the data will no longer be Gaussian but Rician distributed. For an MR magnitude image defined on a discrete grid $\Omega$, $M = \{m_i | i \in \Omega\}$, the probability distribution function of $m_i$, with underlying signal $A$, is (Rice 1945, Bernstein et al 1989)

$$p(m_i|A, \sigma_g) = \frac{m_i}{\sigma_g^2} e^{-\frac{m_i^2}{2\sigma_g^2}} I_0 \left( \frac{Am_i}{\sigma_g^2} \right) \epsilon(m_i),$$

where $I_0(.)$ is the 0th order modified Bessel function of the first kind, $\epsilon(.)$ is the Heaviside step function and $\sigma_g^2$ denotes the variance of the Gaussian noise in the complex MR data. If $A$ equals zero, the Rician PDF simplifies to a Rayleigh distribution (Edelstein et al 1984)

$$p(m_i|\sigma_g) = \frac{m_i}{\sigma_g^2} e^{-\frac{m_i^2}{2\sigma_g^2}} \epsilon(m_i).$$

At high SNR, the Rician PDF approaches a Gaussian PDF with mean $A$ and variance $\sigma_g^2$:

$$p(m_i|\sigma_g) = \frac{1}{\sqrt{2\pi\sigma_g^2}} e^{-\frac{(m_i-A)^2}{2\sigma_g^2}} \epsilon(m_i),$$

i.e. Rician noise in magnitude MR images assumes a Gaussian distribution when the SNR is high and Rayleigh distribution for low SNR.

Proper estimation of the noise variance, $\sigma_g^2$, is important for effective denoising of MRI. Many methods have been proposed in the literature for the estimation of the noise level
from MRI. A survey of these methods is given in Aja-Fernández et al (2009). Most of the noise estimation methods for MRI estimate the noise level from the background area of the magnitude MR image, which is known to be Rayleigh distributed (Sijbers et al 2007, Aja-Fernández et al 2008). These methods need a certain amount of background pixels to perform proper estimation and can fail in the absence of the background region (Aja-Fernández et al 2010). Recently, in Rajan et al (2010), Coupé et al (2010) and Manjón et al (2010), methods are proposed to estimate noise in the absence of background. In Rajan et al (2010), two different approaches based on the local estimation of variance using ML and skewness are proposed and the methods suggested in Coupé et al (2010) and Manjón et al (2010) are based on the correction factor suggested in Koay and Basser (2006). In this work, we followed the methods in Rajan et al (2010) for noise variance estimation.

3. Methods

3.1. Signal estimation using LML

Let \( m_1, m_2, \ldots, m_n \) be \( n \) statistically independent observations from a region of constant signal intensity \( A \). Then, the joint pdf of the observations is

\[
p([m_i] | A, \sigma_g) = \prod_{i=1}^{n} \frac{m_i}{\sigma_g} e^{-\frac{m_i^2 + A^2}{2\sigma_g^2}} I_0 \left( \frac{Am_i}{\sigma_g^2} \right).
\]

(4)

The ML estimate of \( A \) can now be computed by maximizing the likelihood function \( L(A) \) or equivalently \( \ln L(A) \), with respect to \( A \) (Sijbers and den Dekker 2004):

\[
\ln L = \sum_{i=1}^{n} \ln \left( \frac{m_i}{\sigma_g^2} \right) - \sum_{i=1}^{n} \frac{m_i^2 + A^2}{2\sigma_g^2} + \sum_{i=1}^{n} \ln I_0 \left( \frac{Am_i}{\sigma_g^2} \right)
\]

(5)

and

\[
\hat{A}_{ML} = \arg\max_A (\ln L).
\]

(6)

In a region of constant signal amplitude, the true underlying signal can be estimated from the noisy signal using (6). The straightforward approach to denoise MR images using the above-explained ML estimation method is to apply the method locally for each pixel with the assumption that the underlying area is constant for a small region. However, this assumption is not valid for regions with edges and fine structures and as a result these regions will get blurred. This effect can be observed from figure 1. It can also be observed from figure 1(c) and figure 1(d) that the image denoised with a window size of \( 3 \times 3 \times 3 \) is, not surprisingly, less blurred than the one denoised with a window size of \( 5 \times 5 \times 3 \). However, selecting a very small window size is less effective for denoising, especially in smooth areas, since the number of samples for estimation is smaller.

In the proposed method, we consider only the pixels that have an underlying gray-level value close to that of the center pixel in the local window for the true signal estimation, instead of selecting all pixels in the window. This approach can reduce the side effects of direct local ML estimation. However, selection of pixels with similar underlying gray value from a noisy image is a difficult problem. To solve this issue, we used a reference image. The reference image is created by applying the NLM algorithm over the noisy image. Based on the information from the reference image, the corresponding pixels are selected from the noisy image for true signal estimation.
3.2. Nonlocal means algorithm

The NLM method which we used to create the reference image is briefly explained in this section. The NLM method was proposed by Buades et al (2005) and is based on the Markovian hypothesis which states that pixels with a similar neighborhood have a similar gray level value. Given a noisy image \( M = \{m_i | i \in \Omega\} \), where \( \Omega \) represents the image space and \( m_i \) corresponds to the noisy image value at location \( i \), the filtered value at a point \( i \), \( v_i \), is calculated using the NLM method as a weighted average of all the pixels in the image (Buades et al 2005):

\[
v_i = \frac{1}{C_i} \sum_{j \in \Omega_i} w_{i,j} m_j,
\]

where \( \Omega_i \) represents the neighborhood pixels and

\[
C_i = \sum_{j \in \Omega_i} w_{i,j}
\]

is a normalization constant and the weight \( w_{i,j} \) is determined by the similarity of the Gaussian neighborhood between pixels \( i \) and \( j \), which can be expressed as

\[
w_{i,j} = \exp \left( -\frac{\|N_i - N_j\|_2^2}{a^2} \right),
\]

where \( N_i \) denotes a square neighborhood centered at pixel \( i \), \( \| \cdot \|_2^2 \) is a Gaussian-weighted Euclidean distance function, \( a \) is the standard deviation of the Gaussian kernel and \( h \) acts as a degree of filtering.

3.3. Signal estimation using restricted LML

To overcome the drawback of the local ML (LML) estimation method, we propose a restricted LML (RLML) estimation method. In RLML, only the pixels in the local neighborhood of the noisy pixel \( m_i \) that have an underlying gray level value close to the underlying gray level value of \( m_i \), will be considered for the true signal estimation. However, as mentioned earlier, selection of pixels with similar underlying gray value from a noisy image is a difficult problem. To this end, we create a reference image using the above-mentioned NLM method. Now, to denoise a noisy pixel \( m_i \) at \( i \), a list \( l_i \) is created from the neighbors of \( m_i \):

\[
l_i = \{m_j, (j \in \Omega_i) \mid \text{abs}(f(m_j) - f(m_i)) < t\},
\]
where $\Omega$ represents the neighborhood space around $m$, $f(m) = \nu_j$ and $f(m_i) = \nu_i$. The threshold $t$ used for the classification is calculated from the reference image as the range of the intensity dispersion of a uniform area. This can also be automatically computed from the reference image, $\nu$, as the mode of all the local distributions of the range computed around the neighborhood of each pixel:

$$
t = \text{mode} \{ \text{range}(\nu_F)_w \}$$

where $\nu_F$ represents the foreground region of the reference image and $w$ is the neighborhood window size. Contrary to complex-valued images, where the noise is Gaussian distributed, in magnitude images the range of intensity dispersion will depend on the local SNR. Hence, to reduce the error in the classification of pixels in the reference image, the threshold $t$ is computed only from the foreground region of the image. Figure 2 shows the distribution of the local range for a uniform region in the reference image and also the distribution of the local range computed with a neighborhood size of $3 \times 3$ and $5 \times 5$. It can be observed from the plots in the figure that the mode of the distributions of the local range is close to the actual range of the intensity dispersion of the selected homogeneous region. Once a list is created as mentioned in (10), the denoised pixel $\hat{A}_i$ at location $i$ can be computed by substituting the values in the list $l_i$ as the Rician-distributed magnitude data points in (5) and then maximizing the log likelihood function. Applying this procedure to all pixels in the noisy image will give the denoised image. The method is summarized in algorithm 1.

### 3.4. Spatially varying noise levels

When measuring the noise in MR images, it is often assumed that the noise is not spatially varying across the image. Parallel MRI (pMRI) acquisition techniques such as sensitivity encoding (SENSE) or generalized autocalibrating partially parallel acquisitions (GRAPPA) introduce a spatially varying noise variance across the image (Manjon et al. 2010, Aja-Fernandez et al. 2010). To deal with spatially varying noise, local noise variance estimation should be introduced (Manjon et al. 2010). Therefore, we estimate both the noise variance and the underlying true intensity simultaneously by maximizing the log likelihood function in (5) with respect to both $A$ and $\sigma^2$. The ML estimate is then found from the global maximum of $\ln L$ w.r.t. $A$ and $\sigma^2$ (Sijbers and den Dekker 2004):

$$\{ \hat{A}_{\text{ML}}, \hat{\sigma}^2_{\text{ML}} \} = \arg \{ \max_{A, \sigma^2} (\ln L) \}.$$ 

### Figure 2.

Computation of threshold from ROI and from the mode of the local distribution of range: (a) reference image with ROI (marked as red), (b) distribution of intensity in the ROI, (c) distribution of local range computed using $3 \times 3$ and $5 \times 5$ neighborhood.
Algorithm 1. Algorithm for signal estimation using RLML

1: Estimate the noise standard deviation $\sigma_g$ from the input magnitude image $M$ using the method described in Rajan et al. (2010)
2: Create the reference image $\nu$ using (7)
3: Compute the threshold $t$ from $\nu$ by applying (11) using a neighborhood window of size $n \times n \times n$
4: for every pixel $m$ of $M$
5: Create the list $l_i$ as mentioned in (10) using a neighborhood window of size $w \times w \times w$
6: Substitute the values in the list $l_i$ in (5)
7: Estimate $\hat{A}_i$ by maximizing (5) with respect to the unknown true intensity
8: end for

4. Experiments and results

To evaluate and compare the proposed methods with state-of-the-art methods, we did experiments on both synthetic and real MR images. To conduct the experiments on synthetic data, we used the standard MR image phantom of the brain obtained from the Brainweb database (Cocosco et al. 1997) (T1-weighted, intensity values in the range 0–255) and for the experiments on DWI, we simulated a set of DWI of the human brain using the methods in Leemans et al. (2005, 2009). For the experiments on real data, we used the MR images of a kiwi fruit. The proposed algorithm was compared with the following recently proposed methods.

i. NLML: the nonlocal ML method (He and Greenshields 2009) with search window size = $11 \times 11 \times 11$, neighborhood size = $3 \times 3 \times 3$ and sample size 25.
ii. UNLM: unbiased nonlocal means (Manjón et al. 2008) with search window size = $11 \times 11 \times 11$, neighborhood size = $3 \times 3 \times 3$ and the value of the decay parameter $h = \sigma_g$.
iii. RNRAD: noise-driven anisotropic diffusion filter for MRI (Krissian and Aja-Fernández 2009). Local statistics were computed on a $3 \times 3 \times 3$ neighborhood and time step as $dt = 1/6$. For simulations, the number of iterations was chosen as the one with best PSNR when compared with the original image.
iv. ARNLM: adaptive Rician nonlocal means filter with wavelet mixing (Manjón et al. 2010).

For a quantitative analysis of the denoising methods, we used the peak signal to noise ratio (PSNR), the structural similarity index matrix (SSIM) (Wang et al. 2004), Bhattacharyya coefficient (BC) (Bhattacharyya 1943) and the mean absolute difference (MAD). Figure 3 shows the visual quality comparison of the image denoised with NLML, UNLM, RNRAD and the proposed RLML method. This experiment was conducted on a 2D slice of the synthetic image of the brain in the 3D environment after corrupting the image by noise with $\sigma_g = 20$. The proposed RLML filter was executed with the following parameters: neighborhood size for denoising as $7 \times 7 \times 3$ and the neighborhood size for the local computation of the range as $3 \times 3 \times 3$.

In visual analysis, the expectations are as follows: (i) perceptually flat regions should be smooth as possible, (ii) image edges and corners should be well preserved, (iii) texture detail should not be lost and (iv) few or ideally no artifacts (He and Greenshields 2009, Chen et al. 2010). It can be observed from figure 3 that the image denoised with the proposed method is...
closer to the original one (based on the above-mentioned criteria) than the images denoised with the other approaches. Figure 4 shows the quantitative analysis of the proposed method with other recently proposed methods. All the methods to which the proposed method was compared were based on the Rician noise model. In the quantitative analysis, the background was excluded, that is, only the area of the image inside the skull was considered. It can be studied from the plots in figure 4 (taken as a mean of 15 experiments) that the proposed method outperforms other approaches in terms of PSNR, SSIM, BC and MAD.

Figures 5 and 6 show the results of the experiment conducted on the synthetic image of the brain with spatially varying noise which was generated using the method mentioned in Manjón et al. (2010). The noisy images are then denoised with the proposed RLML and also with the recently proposed ARNLM filter. Since the NLM approach is not effective in the case of spatially varying noise, we used the adaptive NLM (ANLM) proposed in Manjón et al. (2010) as the reference image for threshold computation. Both visual and quantitative analyses show that the image processed with RLML is more effective than ARNLM in denoising spatially varying noise.

For the experiments on DW-MRI, we simulated a set of DW images of the human brain with the following parameters: $b = 1200 \text{s mm}^{-2}$, voxel size $= 1.79 \times 1.79 \times 2.4 \text{mm}^3$. 

Figure 3. Denoising MRI with various methods: (a) original image, (b) original image corrupted with Rician noise of $\sigma_g = 20$, (c) part (b) denoised with the NLML method, (d) part (b) denoised with the UNLM method, (e) part (b) denoised with the RNRAD method and (f) part (b) denoised with RLML method.
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Figure 4. Quantitative analysis of the proposed method with other recently proposed methods based on (a) PSNR, (b) SSIM, (c) BC and (d) MAD for image corrupted with Rician noise of $\sigma_g$ varying from 5 to 80.

Figure 5. Denoising MRI with spatially varying noise levels: (a) noisy image with spatially varying noise, (b) noise modulation map, (c) noisy image denoised with the ARNLM method and (d) noisy image denoised with the proposed RLML method.

image size = $107 \times 79 \times 60$ and the gradient orientations = 15. The DW images were then corrupted with Rician noise of $\sigma_g = 100$. The denoised DW images were then generated using the proposed method and other methods mentioned earlier. The fractional anisotropy (FA) map was calculated for the ground truth, the noisy and the denoised methods
Figure 6. Performance comparison of the RLML method against a recently proposed ARNLM method in terms of (a) PSNR, (b) SSIM, (c) BC and (d) MAD for image corrupted with spatially varying Rician noise of $\sigma_g$ varying from 5 to 80.

(see figure 7). Figure 8 shows the absolute difference of the estimated FA map with the original FA map for the noisy and all denoising methods. The MAD of the FA residuals shows that the error in the FA map computed from the image denoised with the proposed method is comparatively less than the other methods.

Figure 9 shows the results of applying the different denoising methods on an MR image of a kiwi fruit. Two sets of kiwi fruit images were reconstructed, one without averaging and the other by averaging 12 acquisitions. Averaging was done in the complex k-space. The denoising algorithms were then applied over the image reconstructed without averaging and the resultant denoised image was compared with the image reconstructed by averaging 12 acquisitions. It can be observed from the images that the visual results are much better for RLML in terms of image contrast. Quantitative analysis of the experiments on the kiwi fruit was done based on the second order moment of the Rice distribution:

$$E[M^2] = A^2 + 2\sigma_g^2.$$  (13)
Figure 7. Experiments on the DWI atlas of the human brain: (a) FA map computed from the ground truth (b) FA map computed after corrupting the original image with Rician noise of $\sigma_g = 100$, (c) FA map computed from the image denoised with the NLML method, (d) FA map computed from the image denoised with the UNLM method, (e) FA map computed from the image denoised with the RNRAD method, (f) FA map computed from the image denoised with the proposed RLML method.

Figure 8. Absolute FA residuals: (a) noisy (MAD: 0.1189), (b) NLML method (MAD: 0.0560), (c) UNLM method (MAD: 0.0531), (d) RNRAD method (MAD: 0.0640), (e) proposed RLML method (MAD: 0.0505).
Figure 9. Experiments on the MR image of a kiwi fruit: (a) original image reconstructed with 1 average, (b) original image reconstructed with 12 averages, (c) part (a) denoised with the NLML method (sample size : 18), (d) part (a) denoised with the UNLM method, (e) part (a) denoised with the RNRAD method and (f) part (a) denoised with the RLML method.

Table 1. Quantitative analysis of the proposed method with other recently proposed methods based on (14). This experiment was conducted on the MR image of a kiwi fruit.

<table>
<thead>
<tr>
<th>Method</th>
<th>Estimated $\hat{\sigma}_g$ based on (14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kiwi fruit with 1 average</td>
<td></td>
</tr>
<tr>
<td>(noise standard deviation = 27.5)</td>
<td></td>
</tr>
<tr>
<td>Kiwi fruit with 12 averages</td>
<td>31.01</td>
</tr>
<tr>
<td>NLML</td>
<td>32.84</td>
</tr>
<tr>
<td>UNLM</td>
<td>38.17</td>
</tr>
<tr>
<td>RNRAD</td>
<td>23.75</td>
</tr>
<tr>
<td>RLML</td>
<td>30.18</td>
</tr>
</tbody>
</table>

If we assume the denoised image $\hat{A}$ as the ground truth, then $\hat{\sigma}_g^2$ can be estimated as

$$\hat{\sigma}_g^2 = \frac{\langle M^2 \rangle - \langle \hat{A}^2 \rangle}{2},$$

where $\langle \rangle$ denotes the spatial average of the whole image. The closer the estimated $\hat{\sigma}_g^2$ to the actual noise variance, the closer the denoised image $\hat{A}$ to the ground truth. The result of this experiment is given in table 1. It is clear from the table that the $\hat{\sigma}_g$ estimated for the image denoised with the proposed RLML method is more close to the actual noise variance. The
estimated standard deviation of the noise from the image acquired without averaging is 27.5. The expected value of $\tilde{\sigma}_g$ (estimated using (14)) after reconstructing an image by averaging multiple acquisitions or by denoising is a value close to 27.5. Among the denoising methods considered, the estimated value of $\tilde{\sigma}_g$ more close to 27.5 is the one denoised with RLML. This experiment on the real data set additionally indicates that the image denoised with the proposed method is more close to the ground truth than other methods.

5. Conclusion

A new method to denoise the MR images by applying the ML method locally to restricted neighborhood is proposed in this paper. A scheme is developed to locally select the appropriate subset of pixels from the neighborhood of each pixel. Through this approach, the side effects of the LML method, the blurring effect and the distortion of fine structures can be reduced. Experiments have been carried out on simulated and real data sets. Quantitative analysis at various noise levels based on the similarity measures, PSNR, SSIM, BC and MAD shows that the proposed method is more effective than other state-of-the-art methods. Experiments were also performed on DW images to prove the efficacy of the proposed method. The MAD of the FA residuals shows that the image denoised with the proposed method is more close to the ground truth.

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